# Cooperative Non-Line-of-Sight Localization Using Low-rank + Sparse Matrix Decomposition

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14. ABSTRACT

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# Cooperative Non-Line-of-Sight Localization Using Low-rank + Sparse Matrix Decomposition

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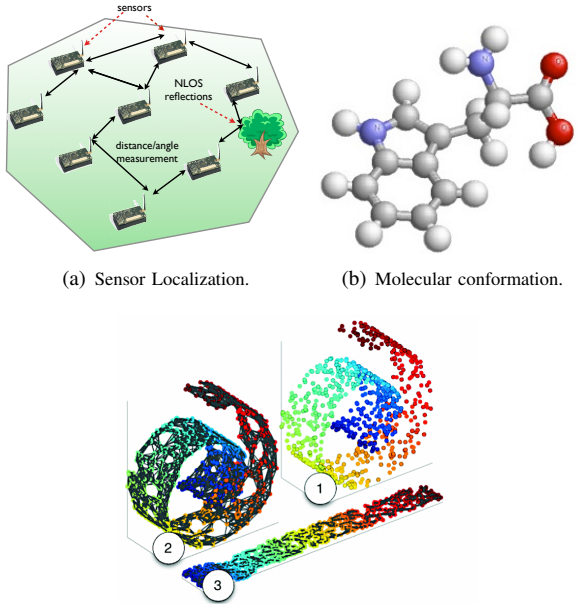
***Abstract*—We consider the problem of estimating the locations of a set of points in a $k$-dimensional euclidean space given a subset of the pairwise distance measurements between the points. We focus on the case when some fraction of these measurements can be arbitrarily corrupted by large additive noise. This is motivated by applications like sensor networks, molecular conformation and manifold learning where the measurement process can induce large bias errors in some fraction of the distance measurements due to physical effects like multipath, spin-diffusion etc. Given the NP-completeness of the problem, we propose a convex relaxation that involves decomposing the partially observed matrix of distance measurements into low-rank and sparse components, wherein the low-rank component corresponds to the Euclidean Distance Matrix and the sparse component is a matrix of biases. Using recent results from the literature, we show that this convex relaxation yields the exact solution for the class of fixed radius random geometric graphs. We evaluate the performance of the algorithm on an experimental data set obtained from a network of 44 nodes in an indoor environment and show that the algorithm is robust to non-line-of-sight bias errors.**

***Keywords:*** Non-Line-of-Sight localization, robust matrix decomposition.

## I. Introduction

The problem of obtaining the locations of a set of points given pairwise distances between the points is a topic of significant research interest. The problem has applications in a broad spectrum of areas such as sensor networks, molecular biology, data analysis, manifold learning etc. In sensor networks, the locations of different sensor nodes need to be estimated, given the distance measurements between nodes that are within some communication radius of each other [1] (Fig. 1(a)). The structure of a protein molecule is determined by estimating the distances between the component atoms of the molecule using techniques such as NMR spectroscopy [2] (Fig. 1(b)). Many applications that involve processing massive data sets in high dimensions require efficient representations of the data in a low dimensional space. Most of these data sets tend to span a low dimensional hypersurface in the higher dimensional space. Pairwise proximity measurements between the data points could be used to obtain an efficient representation of the data in lower dimensions preserving the relative conformation of the points [3] (Fig. 1(c)).

Given the range of applications, significant research work is devoted in the literature dealing with theory and algorithms focusing on localization tailored to the problem of interest. The problem is shown to be NP-complete [4] in the general case wherein one is provided with an arbitrary subset of the pairwise distance measurements and is asked to find a valid configuration of points satisfying these distance measurements. This is hard even when one has the side-information that there is a unique set of points that satisfy the given distances. Hence, most of the work in the literature focus on developing efficient localization algorithms with provable theoretical guarantees for specific node geometries. This is when the distance measurements are either exact or slightly perturbed, which is the case for line-of-sight (LOS) localization [5], [6]. Existing theoretical results [7], [6] have shown that LOS localization can be achieved in polynomial time for random geometric graphs.




(a) Sensor Localization. (b) Molecular conformation.

(c) Manifold Learning.

Figure 1. (a) Sensors placed in a field for monitoring the region. (b) Tryptophan, one of the 20 standard amino acids [8]. (c) (1) A high dimensional "swiss roll" data set that spans a lower dimensional hypersurface [9]. (2) Data points within a local radius are connected. (3) The lower dimensional representation of the high dimensional surface preserving the relative geometry of the points.

We consider a generalized version of the original LOS

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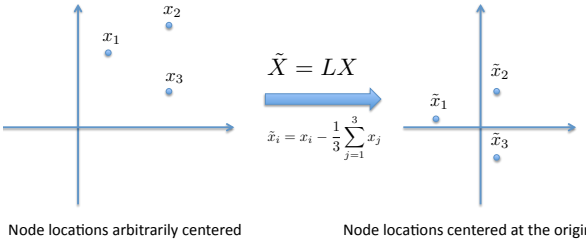



Figure 2. Centering the node locations.

$$
\begin{aligned}
D &= \begin{bmatrix} 0 & ||x_1-x_2||^2 & ||x_1-x_3||^2 \\ ||x_1-x_2||^2 & 0 & ||x_2-x_3||^2 \\ ||x_1-x_3||^2 & ||x_2-x_3||^2 & 0 \end{bmatrix} \\
&= \begin{bmatrix} ||x_1||^2 & ||x_1||^2 & ||x_1||^2 \\ ||x_2||^2 & ||x_2||^2 & ||x_2||^2 \\ ||x_3||^2 & ||x_3||^2 & ||x_3||^2 \end{bmatrix} + \begin{bmatrix} ||x_1||^2 & ||x_2||^2 & ||x_3||^2 \\ ||x_1||^2 & ||x_2||^2 & ||x_3||^2 \\ ||x_1||^2 & ||x_2||^2 & ||x_3||^2 \end{bmatrix} - 2 \begin{bmatrix} ||x_1||^2 & x_1^T x_2 & x_1^T x_3 \\ x_2^T x_1 & ||x_2||^2 & x_2^T x_3 \\ x_3^T x_1 & x_3^T x_2 & ||x_3||^2 \end{bmatrix} \\
&= \begin{bmatrix} ||x_1||^2 \\ ||x_2||^2 \\ ||x_3||^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} ||x_1||^2 & ||x_2||^2 & ||x_3||^2 \end{bmatrix} - 2 \begin{bmatrix} ||x_1||^2 & x_1^T x_2 & x_1^T x_3 \\ x_2^T x_1 & ||x_2||^2 & x_2^T x_3 \\ x_3^T x_1 & x_3^T x_2 & ||x_3||^2 \end{bmatrix} \\
&= \underbrace{vecdiag(XX^T)}_{\text{column vector of diagonal entries}} \mathbf{1}^T + \mathbf{1} vecdiag(XX^T)^T - 2XX^T
\end{aligned}
$$

Figure 3. Relating the distance matrix and the node locations.

localization problem, wherein we assume that some fraction of the distance measurements can have large bias errors. This problem is traditionally known as non-line-of-sight (NLOS) localization. The motivation for this problem setup arises from real world physical processes that cause bias errors in the measurements. For example, multipath interference causes huge bias errors in the distance measurements obtained between nodes in a sensor network or from the GPS satellites in vehicular networks [10]. The spin-diffusion phenomenon in NMR spectroscopy causes apparent shrinkage of the distance measurements in protein molecular conformation applications. Outliers in the data can cause large bias errors in applications like manifold learning. Thus there has been significant recent interest in developing algorithms that tackle NLOS errors [10], [11]. However, there are no theoretical guarantees for this problem to the best of our knowledge.

There is some existing work in the literature that formulates LOS localization as a low-rank matrix completion problem [12]. The problem boils down to completing a low-rank matrix of squared distances known as the Euclidean Distance Matrix (EDM) and there are efficient convex relaxations for this [9]. Our contribution is the following:

(a) We formulate the NLOS localization problem as one of matrix decomposition wherein we are interested in decomposing the partially observed matrix of corrupted pairwise squared distances into a low rank EDM and a sparse matrix of bias errors.
(b) Using existing results from the matrix decomposition literature, we show that the proposed relaxation will achieve exact localization for a random geometric graph with radius greater than a threshold that is a function of the fraction of LOS measurements.
(c) We evaluate the algorithm performance on a real-world dataset obtained from an indoor network of 44 nodes.

## II. Problem Setup

Consider the case where we have $N$ points/nodes located inside a unit cube in the $k$-dimensional euclidean space. Let $X \in \mathbb{R}^{N \times k}$ be the vector of all node locations, where each row represents a point in the $k$-dimensional space. Let us assume that these points are placed uniform randomly in this space and we have distance measurements between any two points that are within a radius $r$ of each other. Note that given only these distance measurements, one can at most hope to obtain the conformation of points congruent to rotations and translations. Let $D \in \mathbb{R}^{N \times N}$ be the matrix of all possible pairwise squared distances, i.e. $D_{ij} = ||x_i - x_j||^2$, where $x_i^T$ is the $i$th row of $X$.

Let $\hat{D}$ be the matrix of distance measurements that is partially observed. We will assume the following measurement model,

$$\hat{D}_{ij} = D_{ij} + B_{ij} + N_{ij} \quad \forall\ (i,j) \mid ||x_i - x_j|| \leq r,$$

where $B_{ij}$ is the bias in the measurement due to NLOS errors and $N_{ij}$ is the thermal noise. $B$ is taken to be the matrix of biases. We will assume that only some fraction ($\alpha$) of the entries $\{B_{ij},\ (i,j) \mid ||x_i - x_j|| \leq r\}$ are non-zero. Further we will also assume that the thermal noise is a small perturbation that is bounded by a quantity $\Delta$.

We need to relate the matrix $D$ and the node locations $X$. Clearly, $D$ is invariant to rigid transformations of $X$, i.e. rotations, translations and reflections of the relative node placements. Hence we need to define a metric for comparing node location estimates that is invariant to these rigid transformations. The following equation relates the distance matrix $D$ and the node locations $X$, $D = vecdiag(XX^T)\mathbf{1}^T + \mathbf{1}vecdiag(XX^T)^T - 2XX^T$, where the $vecdiag$ operation creates a column vector of the input matrix by stacking the diagonal entries and $\mathbf{1}$ is a vector of ones (see Fig. 3). We will now derive one possible set of node locations given a distance matrix $D$. Since $D$ is invariant to translations, let us normalize the node locations so that they are centered at the origin. Define $L = I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T \in \mathbb{R}^{N \times N}$. Note that $\tilde{X} = LX$ is a set of node locations that are centered at the origin (see Fig 2). Since $\mathbf{1}$ lies in the left and right null spaces of $L$, by multiplying the expression for $D$ on both sides by $L$, we get that $-\frac{1}{2}LDL = LX(LX)^T$. Thus given a fully observed noiseless distance matrix $D$, one can obtain a set of node locations centered at the origin by choosing $\tilde{X} = U\Sigma^{\frac{1}{2}}$, where $-\frac{1}{2}LDL$ has the singular value decomposition (SVD), $-\frac{1}{2}LDL = U\Sigma U^T$. In order to compare two possible node location estimates, $X$ and $\hat{X}$, we will use the following error metric $||LXX^TL - L\hat{X}\hat{X}^TL||$. Note that $LX$ takes care of the translations and $XX^T$ is invariant to rotations and reflections.

With this background we can now proceed with the problem formulation. Note that the rank of $D \in \mathbb{R}^{N \times N}$ is at most $k+2$ irrespective of the number of nodes $N$. Thus the problem of interest is to decompose the partially observed measurement matrix $\tilde{D}$, as a low-rank matrix $D$ and a sparse matrix $B$. The reason to look for a sparse matrix B is in the hope that only a small fraction of the measurements are arbitrarily corrupted and hence we want a configuration of points with the

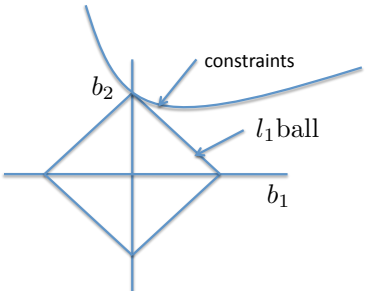



Figure 4. L1 approximation to the L0 norm

**Algorithm 1** Matrix decomposition for NLOS localization

1: **Input:** distance measurements $\tilde{d}_{ij}$, $(i,j) \in \mathcal{E}$, dimension $k$, bound on the measurement noise $\Delta$.

2:

$$(\hat{D}, \hat{B}) = \arg\min ||D||_* + \lambda ||vec(B)||_1$$
$$s.t. \quad ||D_{ij} + B_{ij} - \tilde{D}_{ij}|| \leq \Delta \quad \forall (i,j) \in \mathcal{E}$$

3: Compute the rank $k$ approximation of $-\frac{1}{2} L \hat{D} L^T$ and its SVD as $\hat{U}_k \hat{\Sigma}_k \hat{U}_k^T$.

4: **Output:** $\hat{X} = \hat{U}_k \hat{\Sigma}_k^{\frac{1}{2}}$.

minimum number of distance measurements being corrupted by bias errors. Thus in the noiseless case ($\Delta = 0$), we have the following,

$$\begin{aligned} \text{find} \quad & (D, B) \\ \text{s.t.} \quad & \tilde{D}_{ij} = D_{ij} + B_{ij} \quad \text{for all observed distances,} \\ & D \text{ is of rank at most } k+2, \quad B \text{ is sparse.} \end{aligned}$$

This problem is non-convex and NP-hard in general for arbitrary low-rank matrices. This is because, the rank constraint on $D$ essentially says that the L0-norm of the singular values of $D$ should be less than or equal to $k+2$ which is a highly non-convex constraint and NP-hard to solve. Similarly, the sparsity constraint on $B$ would require us to minimize the L0-norm of the matrix $B$ which is also NP-hard. Hence in order to have a tractable optimization problem, we will look for the closest convex approximations to these two constraints that is easier to solve. The smallest convex ball approximation to the L0-norm is the L1-norm. An example of this is shown in Fig. 4. The L0-norm corresponds to the points where the lines intersect the two axes since one of the coordinates is zero and hence these form the contours of this norm. In solving the optimization problem, the constraints on the observed distance measurements essentially manifest as a plane in this dimension or in general a convex surface (see Algorithm 1). Thus the solution is the point where the smallest L1-ball touches this surface. Similarly, a convex approximation for the rank constraint, is the L1-norm of the singular values of $D$ known as the nuclear norm. Thus we will use the nuclear norm and the L1-norm as surrogates for rank and sparsity respectively. These are standard convex relaxation techniques explored in the low-rank matrix completion literature [13]. Algorithm 1 details the optimization problem that we will solve. This problem of decomposing a matrix into a low-rank and a sparse matrix has been of recent interest in the literature [13] and we use existing results to provide theoretical guarantees. We will focus on the case when $\Delta = 0$.

## III. Results

The following theorem summarizes our result.

**Theorem 1.** *For a random placement of nodes in $[-1,1]^k$, ignoring border effects, as long as the radius of connectivity $(r)$ between nodes that obtain pairwise distance measurements satisfies $r \geq O\left(\left(\frac{1}{1-\alpha}\right)^{\frac{1}{k}}\right)$ where $\alpha$ fraction of the measurements have bias errors, the proposed optimization routine recovers the node locations exactly modulo rigid transformations with high probability.*

*Proof:* Standard matrix decomposition into low-rank and sparse components can be achieved only when the low-rank matrix is also not sparse. For this, the singular vectors of the matrix have to be well spread out for which they need to satisfy certain conditions known as *incoherence* [13] conditions. In our case, as the singular vectors are strongly related to the node location vectors, this condition is equivalent to requiring that the node locations be well spread out which is true for a random node placement. Further, if we have to reconstruct every row of the low-rank matrix, then we need to observe at least some fraction of the entries in that row that are uncorrupted. This places a lower bound on the number of neighbors each node has and hence a restriction on the radius of connectivity which would be a function of the fraction of corrupted entries. Thus the proof ingredients involve showing that the EDM is incoherent with high probability for a random node placement and obtain a lower bound on the radius of connectivity.

We will first show that the EDM is incoherent with high probability for a random placement of nodes. The proof is modeled closely on the lines of the results of Oh et. al. [14] with slight changes needed to account for the modified incoherence definitions of Chen et. al. [13]. The restrictions on the number of observed corrupted entries in each row would impose a lower bound on the radius of connectivity between the nodes. Our proofs here are shown to hold true in expectation and these can be converted to high probability results using concentration inequalities.

Given the true locations $X$ and the estimated locations $\hat{X}$, we will use the distortion metric $||LXX^TL - L\hat{X}\hat{X}^TL||$, to characterize how close the estimated relative node geometry is with respect to the true node geometry. The norm can be taken to be the Frobenius norm for simplicity. Since matrix recovery results would characterize the error in recovering the

distance matrix $D$, the following lemma would be useful,

**Lemma 1.** $||LXX^TL - L\hat{X}\hat{X}^TL|| \leq ||D - \hat{D}||$.

*Proof:*

$$\begin{aligned} ||L(XX^T - \hat{X}\hat{X}^T)L|| &\leq ||L(XX^T + \frac{1}{2}\hat{D})L|| \\ &+ ||L(-\frac{1}{2}\hat{D} - \hat{X}\hat{X}^T)L||, \\ &= \frac{1}{2}||D - \hat{D}|| \\ &+ ||-\frac{1}{2}L\hat{D}L - L\hat{X}\hat{X}^TL||. \end{aligned}$$

Note that $L\hat{X}\hat{X}^TL$ is the best rank $k$ approximation to $-\frac{1}{2}L\hat{D}L$. Thus if $S$ is any other rank $k$ matrix, we have that $||-\frac{1}{2}L\hat{D}L - S|| \geq ||-\frac{1}{2}L\hat{D}L - L\hat{X}\hat{X}^TL||$. Choosing $S$ to be $-\frac{1}{2}LDL$ we get that,

$$||L(XX^T - \hat{X}\hat{X}^T)L|| \leq ||D - \hat{D}||.$$

■

Given the above lemma, we can now concentrate on bounding the error $||D - \hat{D}||$. We will use the following result of Chen et. al. [13]. Consider matrices $R$ and $S \in \mathbb{R}^{N\times N}$ such that $R$ is low rank and $S$ is sparse (the exact conditions for low rank and sparsity will be given in the theorem). Given a subset $\Omega$ of the entries of $\tilde{R} = R+S$, we are interested in recovering $R$ and $S$. Assume that $R$ and $S$ are symmetric. $R$ has a SVD decomposition $U\Sigma U^T$. $R$ is said to be $(\mu, k')$ incoherent if the following conditions hold for some $k' \in \{1, ..., N\}$ and $\mu \in \{1, ..., \frac{N}{k'}\}$.

- rank(R) $= k'$.
- $\max_i ||U^T e_i|| \leq \sqrt{\frac{\mu k'}{N}}$.
- $||UU^T||_\infty \leq \sqrt{\frac{\mu k'}{N^2}}$,

where $e_i$ are the standard basis vectors. Consider the following optimization problem,

$$\begin{aligned} (\hat{R}, \hat{S}) &= \arg\min ||R||_* + \lambda||S||_1 \\ \text{s.t. } (R+S)_{ij} &= \tilde{R}_{ij} \quad \forall (i,j) \in \Omega. \end{aligned}$$

Let $q$ be the sum of the maximum number of unobserved entries and the entries with errors (i.e. $S_{ij} \neq 0$) in any row of $\tilde{R}$. Let $\sigma_{max}(S) \leq \eta q ||S||_\infty$, where $\sigma_{max}$ is the maximum singular value. Let $\beta = 2\sqrt{\frac{\mu k' q}{N}}$. The following theorem holds.

**Theorem 2.** *[13] For* $\lambda \in \left[\frac{1}{1-2\beta}\sqrt{\frac{\mu k'}{N^2}}, \frac{1-\beta}{\eta q} - \sqrt{\frac{\mu k'}{N^2}}\right]$*, there exists a constant* $c$ *independent of* $N$*,* $\mu$ *and* $k'$ *such that with probability* $1 - cN^{-10}$*, the optimization recovers* $(R, S)$ *exactly if*

$$\sqrt{\frac{\mu k' q}{N}}\left(2 + \eta\sqrt{\frac{q}{N}}\right) \leq \frac{1}{2}.$$

In order to apply the above theorem to our problem setting, we need to show that $D$ satisfies the incoherence properties and identify parameters and thereby conditions on the radius $r$ and the fraction $\alpha$ for which the theorem holds. The following lemma states the incoherence of $D$.

**Lemma 2.** $D$ *is* $(16k^2(k+2), k+2)$ *incoherent with high probability.*

*Proof:* Appendix. ■

We need a bound on the maximum singular value of the symmetric bias matrix $B$. We have $\sigma_{max}(B) = \sqrt{\max_z \frac{||Bz||^2}{||z||^2}}$. Let $\Delta'$ be the maximum value of the bias. Using the above equation we get that, $\sigma_{max}(B) \leq \sqrt{N\Delta'^2} \leq \eta q \Delta'$. $\eta$ can thus be chosen as $\frac{\sqrt{N}}{q}$ to satisfy one of the conditions of Theorem 2.

We need to find the number of corrupted and unobserved entries ($q$) in each row. Ignoring border effects, the average number of neighbors of a node in k-dimensional Euclidean space is given by $\kappa r^k N$, where $\kappa = \frac{\pi^{\frac{k}{2}}}{\frac{\Gamma(\frac{k}{2}+1)}{2^k}}$. Assuming that $\alpha$ fraction of the measurements are corrupted by NLOS noise, we have the average number of unobserved entries in every row and the entries with errors given by $\mathbb{E}(q) = N(1-(1-\alpha)\kappa r^k)$. One can show that $q$ is concentrated around its mean and can use existing results from literature [15] to bound this value with high probability. However we omit the details for sake of brevity. Therefore, for the condition in the theorem to be satisfied, we need the following.

$$\sqrt{\frac{\mu k' q}{N}}\left(2 + \eta\sqrt{\frac{q}{N}}\right) \leq \frac{1}{2},$$

$$\sqrt{\frac{\mu(k+2)N(1-(1-\alpha)\kappa r^k)}{N}}\left(2 + \frac{1}{\sqrt{q}}\right) \leq \frac{1}{2}.$$

For large values of $N$ we get that, $r \geq O\left(\left(\frac{1}{1-\alpha}\right)^{\frac{1}{k}}\right)$. ■

## IV. Validation

In this section we provide simulation results and validation on experimental data to evaluate the performance of the proposed algorithm based on robust matrix decomposition.

### A. Simulations

For simulations, we have 50 nodes uniform randomly placed in a grid $[-1, 1]^2$, where the units are taken in meters ($m$). The NLOS bias in the measurements is taken to be a uniform random variable on a support of $[0, 6]$ and the gaussian noise in the measurements is taken to have a standard deviation of $0.02m$. Fig. 5 shows the results for the case where nodes that are within a radius of $1.5m$ of each other obtain distance measurements. The figure is a plot of the normalized error $\frac{||LXX^TL - L\hat{X}\hat{X}^TL||_2}{||LXX^TL||_2}$ as a function of the fraction of NLOS measurements. The plots are obtained after averaging over multiple iterations. One can see that the errors are quite large even when the fraction of NLOS measurements are small.

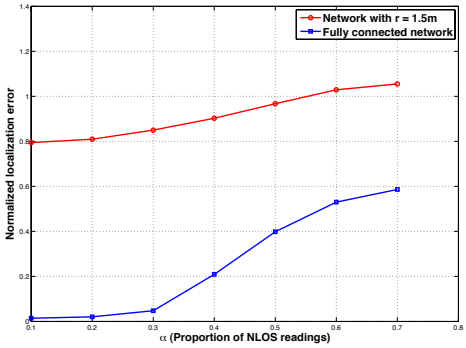



Figure 5. Localization error as a function of the fraction of NLOS measurements ($\alpha$) for a network with $r = 1.5m$ and for a fully connected network.

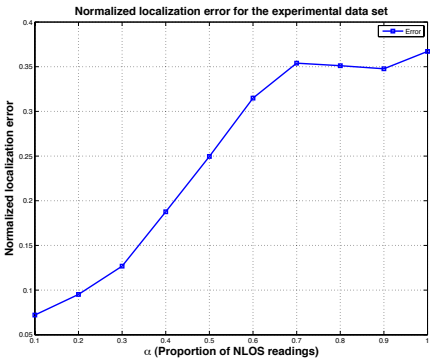
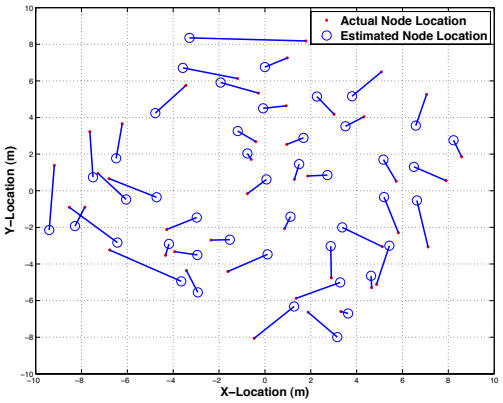
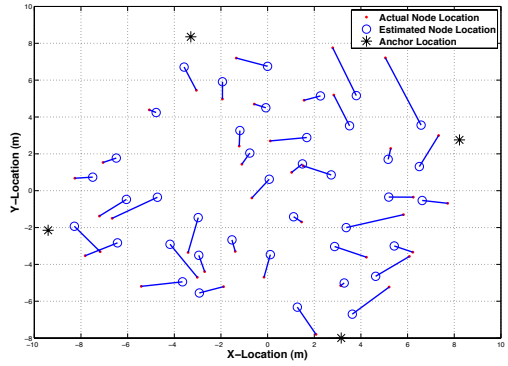



Figure 6. Normalized localization error for the experimental data set as a function of the fraction of corrupted measurements.




(a) Location estimate before rotation.




(b) Location estimate after rotation.

Figure 7. Node location estimates for the experimental data set. (a) Without using anchors. Error per node $= 1.91m$. (b) After rotation by fixing four of the nodes as anchors. Error per node $= 1.47m$.

The theoretical lower bound on the radius itself turns out to be quite large for this case. Even in the LOS case, we carried out simulations for a large network of nodes using fast matrix completion solvers and saw that the localization was not accurate unless the connectivity radius was quite large. Since we are looking at local measurements, our observations are largely biased towards the smaller entries of the low-rank distance matrix which seems to be adversely affecting the estimation process. Fig. 5 also shows the same plot for a fully connected network, i.e. all the pairwise distances are observed and the errors are only due to NLOS propagation. One can see that the algorithm can tolerate up to $30\%$ of the measurements being NLOS.

### *B. Experimental results*

The algorithm was applied to a data set [16] obtained from pairwise measurements in a 44 node network. The experiments were conducted in an indoor office environment by simulating 44 node locations using a transmitter and a receiver and obtaining pairwise time-of-arrival measurements. The environment had a lot of scatterers and nearly all the distance estimates have strong NLOS biases in them. In order to validate the performance as a function of the fraction of NLOS measurements, some fraction of the measured distances were replaced by the true distances and the performance is plotted as function of this fraction in Fig. 6. The true and estimated node locations for the experimental data set (all the measurements are corrupted in this case) is shown in Fig. 7(b). Four nodes were designated as anchors (known node locations) and the other node locations were rotated to match the locations of the anchors in order to have a fair comparison with the true node locations. All the node locations are centered to the origin. The average node error is $1.47m$. This can be contrasted with the error of $1.26m$ reported by Patwari et. al. [16]. However they obtained the node locations after subtracting out the NLOS bias in the measurements and using one of the best LOS localization algorithm. We can see that our algorithm is quite robust since our errors are not too far off given that we work on the NLOS corrupted data. Thus the robust matrix decomposition approach seems to handle the NLOS errors quite well.

## V. Conclusion

In this work, we proposed a low-rank + sparse matrix decomposition framework for the problem of cooperative NLOS localization. Given that the problem is NP-complete in the general case, we showed that it can be solved in polynomial time for the special case of fixed radius random

geometric graphs. We applied the algorithm to a real world sensor measurement dataset and showed that it is robust to NLOS errors. However, we saw from simulations that the connectivity radius needs to be quite large in order to obtain a good estimate of the node locations. This seems to be partially due to the observed entries being largely skewed towards the smaller entries of the distance matrix. One way of tackling this could be to introduce heterogenous nodes in the network that have a larger communication radius and hence we get to observe some fraction of the larger distances. Exploring these possibilities is part of future work.

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## APPENDIX

### A. Proof of Lemma 2 [14]

Let $\tilde{X} = \begin{bmatrix} 1 & x_1^T & ||x_1||^2 \\ 1 & x_2^T & ||x_2||^2 \\ \dots & & \\ 1 & x_N^T & ||x_N||^2 \end{bmatrix} = \begin{bmatrix} \tilde{x}_1^T \\ \tilde{x}_2^T \\ \dots \\ \tilde{x}_N^T \end{bmatrix} \in \mathbb{R}^{N \times k+2}$ and $Y = \begin{bmatrix} 0 & \mathbf{0}^T & 1 \\ \mathbf{0} & -2I_k & \mathbf{0} \\ 1 & \mathbf{0}^T & 0 \end{bmatrix} \in \mathbb{R}^{k+2 \times k+2}$. We have the relation, $D = \tilde{X} Y \tilde{X}^T$. For a random placement of nodes we have that rank $(\tilde{X})$ is $k+2$ with high probability and hence the rank of $D$ is also $k+2$ with high probability.

We can write $\tilde{X} = VA$, where $V \in \mathbb{R}^{N \times k+2}$, $V^T V = I$ and $A \in \mathbb{R}^{k+2 \times k+2}$ (e.g. using $QR$ factorization). Thus we can write $D = VAYA^T V^T$. Let the SVD of $D$ be given as $D = U \Sigma U^T$, $U \in \mathbb{R}^{N \times k+2}$ and $U^T U = I$. Hence $U$ can be expressed as $U = VQ$ for some $Q \in \mathbb{R}^{k+2 \times k+2}$ such that $Q^T Q = I$.

Let $U = \begin{bmatrix} u_1^T \\ u_2^T \\ \dots \\ u_N^T \end{bmatrix}$ and $V = \begin{bmatrix} v_1^T \\ v_2^T \\ \dots \\ v_N^T \end{bmatrix}$. We have $u_i^T = v_i^T Q$. Thus

$$\max_i ||U^T e_i|| = \max_i ||u_i|| = \max_i ||v_i||.$$

We have $V = \tilde{X} A^{-1}$. Thus $v_i^T = \tilde{x}_i^T A^{-1}$. Therefore,

$$\begin{aligned} ||v_i|| &\leq (\sigma_{min}(A^T))^{-1} ||\tilde{x}_i||, \\ &\leq (\sigma_{min}(A^T))^{-1} \sqrt{k(k+2)}. \end{aligned}$$

We have$\sigma_{min}(A^T) = \sqrt{\lambda_{min}(A^T A)}$. Note that $\tilde{X}^T \tilde{X} = A^T A$. Thus we get,

$$\begin{aligned} \tilde{X}^T \tilde{X} &= \sum_{i=1}^{N} \begin{bmatrix} 1 & x_i^T & ||x_i||^2 \\ x_i & x_i x_i^T & x_i ||x_i||^2 \\ ||x_i||^2 & x_i^T ||x_i||^2 & ||x_i||^4 \end{bmatrix} \\ &\rightarrow \begin{bmatrix} N & \mathbf{0}^T & \frac{Nk}{3} \\ \mathbf{0} & \frac{N}{3} I_k & \mathbf{0} \\ \frac{Nk}{3} & \mathbf{0}^T & N\left(\frac{k}{5} + \frac{k(k-1)}{9}\right) \end{bmatrix}. \end{aligned}$$

One can easily verify that $\lambda_{min}(A^T A) = \frac{N}{3}$ with high probability for any value of $k$. Therefore,

$$||v_i|| \leq \sqrt{\frac{4k(k+2)}{n}}.$$

Let us now look at $||UU^T||_\infty$.

$$\begin{aligned} ||UU^T||_\infty &= \max_{ij} |u_i^T u_j|, \\ &= \max_i |u_i^T u_i|, \\ &= \max_i ||v_i||^2, \\ &\leq \sqrt{\frac{16k^2(k+2)^2}{N^2}}. \end{aligned}$$

Thus by choosing $\mu = 16k^2(k+2)$ we get the result.